Teaching Portfolio

Donovan Snyder

Contents

1 Teaching Philosophy Statement

At the heart of my teaching philosophy is the belief that my role as an educator, whether as an instructor, course coordinator, or mentor, is to ensure that learning is approachable. My job is not only to deliver material and hope that students can learn, but to guide everyone, regardless of their background or circumstances, in navigating new information with confidence. Approachability manifests in structural, intellectual, and personal elements, which I detail below. My dedication to this principle has been recognized through the Edward Peck Curtis Award for Excellence in Teaching as a Graduate Student in 2025.

Structuring a Course to Be Approachable

As an instructor and course coordinator, I design the structure of my courses to be as integrated as possible, making every aspect of learning more approachable. For example, when I taught Foundations of Calculus, I added low-stakes written homework to complement the existing online assignments. Previously, the class only used the online assignment system WeBWorK, and while the answer is graded instantly with this system, the questions are not in the style of the exam and there is no feedback on the process of arriving at the answer. With the new written questions, the students get practice applying their knowledge in a low-stakes context similar to the exam format, making the high-stakes assessments more approachable. In anonymous mid-semester feedback I collected, one student said "My favorite is the written section... because I get to work through an entire problem."

To increase the effectiveness of the WeBWorK component I was awarded funding for a Student Course Development Project. We updated and created new assignments to replace a less focused collection of problems and directly aligned them to the course learning outcomes. We also incorporated pedagogical research in the creation of the sets and removed distractions (like notational differences and varying formatting) that increase struggle and cognitive load in ways that do not contribute to learning. We will be monitoring and reporting on the implementation of this project during the semester.

Making Content Intellectually Approachable

Ensuring that students from all backgrounds can truly engage with the material is one of the most important ways to make learning approachable. I begin each of my classes with "Get Started" questions for students to work on in groups. Active learning is often used as a buzzword, but it truly is effective in the classroom, and starting each session in this way immediately puts everyone in a problem solving mentality. Throughout the rest of the class, I give time for students to work on questions and get immediate feedback on their understanding by using free online resources.

I am fortunate enough to have been an instructor of record many times as a graduate student, and this has allowed me to improve the pacing, organization, and material of my courses. A student in my Precalculus class said in end-of-semester feedback that I "did a great job covering the content in this course. He broke things up in a way that was easy to understand and did well at explaining difficult topics while allowing us to practice concepts as well." When I lead learning in other contexts, like honors courses or mentoring students in reading groups, my preparation similarly consists of considering how to make the material approachable.

Over the last five summers, I have taught Precalculus as part of the Early Connections Opportunity (ECO) program for incoming first-year students. I have served as Head Math Instructor for three years which means that, in addition to my instructor duties, I organize the math instructional team, work with other faculty members across the program to help align our efforts, and facilitate advising and mentoring our students. This role has given me valuable experience working with a diverse population of students, many of whom come from underserved backgrounds. I have learned to adapt my teaching strategies to be approachable to a wide range of academic preparedness and learning styles, and to foster an inclusive environment where all students feel supported and empowered to succeed.

Making Education Personally Approachable

Making class meetings approachable is one of the most prominent and important ways to affect student learning. This must start on the first day: I ask for students' names and background (including how they want to be addressed) to immediately begin fostering a more personal connection. Other components of the course structure and syllabus play a large role. To make participation easier and more comfortable, I hold office hours in campus libraries and online, and I will continue to work on ways that I can better reach students. On the syllabus, I clearly link to the library's copy of the standard textbook and provide open education resources like OpenStax's textbooks as alternatives for the students to make the cost of the course less of an obstacle. For this work, in collaboration with our math librarian, I was recognized as a Zero Cost Hero in 2023.

In courses where formal student feedback is not collected, I administer anonymous surveys during and after each course. Combined with official evaluations, this enables me to identify areas for immediate improvement and to make informed adjustments for future iterations. For instance, I have found that the start of my ECO course struggles to balance between effectively challenging students while providing appropriate remediation, given their varied backgrounds. I am currently developing new activities to support the students' differing needs. Additionally, I am collaborating with the instructors of ECO's Academic Skills and Techniques course to design an assignment that helps students build and apply study skills they can use throughout their careers.

Focusing on approachability in these ways allows students to more thoroughly and deeply engage with the material, as opposed to lowering standards or making a course easier. As I continue in my career, I aim to expand my use of inclusive strategies in class, develop better materials for standard and advanced courses, and continue to mentor students outside of class. I am excited to learn and collaborate with colleagues across disciplines, and I look forward to the challenge of leading new courses in approachable ways.

2 Teaching Responsibilities & Experience

2.1 Courses Taught

- Math 140 - Foundations of Calculus
  - College Algebra and Trigonometry
  - Fall 2024
  - Course coordinator of 1 section
  - Responsibilities:
    - Adapt and improve syllabi and course materials, adding low-stakes written homework
    - Manage team of teaching assistants, improve upon workshop materials
    - Manage WeBWorK assignments, Learning Management System
    - Create and grade exams
    - Field and implement mid-semester feedback
- Math 120 - Precalculus Module, for Early Connections Opportunity
  - College Algebra and Trigonometry
  - Summer 2021 - 2025
  - Instructor, Head Math Instructor (2023 - 2025)
  - Responsibilities:
    - Design course curriculum and syllabus for 4 week intensive summary course
    - Mentor students individually to improve performance in course and make academic plans for the semester
    - Oversee other instructors to ensure responsibilities are being met
    - Collaborate with other faculty and staff to ensure scholars are meeting expectations and to provide additional support as needed
    - Currently collaborating with Academic Strategies and Techniques instructors to design new cross-disciplinary assignments
- Math 143 - Calculus III
  - Sequences, series, parametric equations and polar coordinates
  - Summer 2022 - 2025, Spring 2024 - 2025
  - Instructor, Course coordinator of 3 sections (Summer 2025)
  - Responsibilities:
    - Rewrite and improve workshop materials
    - Manage WeBWorK assignments
    - Create and grade exams

2.2 Mentoring

Starting in Fall 2022, I have organized the Graduate Student Reading Group Program, where I facilitate the creation and running of reading courses led by graduate students for undergraduates. Over 20 undergraduates have participated in this extracurricular.

As a part of the program I've mentored the following undergraduates:

- Stuti Shah: Hopf algebras and their representation theory, using Kassel's *Quantum Groups*
- Anastasia Chen, Seungkyo Jeong, and Lily Testa: VC-dimension, using written notes
- Allen Shao: Iterated function systems, using Michael Barnsley's *Fractals Everywhere*
- Vincent Chen: Integral probability, using Borodin and Gorin's *Lectures on Integrable Probability*

I have mentored and advised students through the Early Connections Opportunity program, see above.

2.3 Professional Development

- Through the Teaching Center, book club *Inclusive Teaching: Strategies for Promoting Equity in the College Classroom*, Hogan and Sathy
- Through the Teaching Center, book club *Teaching Matters: A Guide for Graduate Students*, Haynie and Song
- Trainer for Graduate Teaching Assistant Workshop for incoming graduate students

2.4 Contributions to Curriculum Development

For the courses I've been most involved with, MATH 143 and MATH 140, I've rewritten and improved workshop materials. One reason was to add more illuminating and important examples that cover all parts of the learning outcomes for that week, as the previous materials would sometimes skip significant topics, leaving no opportunities for the students to practice. Another reason to rewrite the worksheets was to improve the flow of the workshop, allowing the undergraduate and graduate teaching assistants to better run a workshop that truly allows the students to work on the problems instead of a lecture disguised as a workshop.

Additionally, using funding from the Teaching Center, I am overseeing a Student Course Development Project for MATH 140 in Summer 2025. We are updating and creating new WeBWorK assignments, aligning them directly with the course learning outcomes, and removing notational differences and varying formatting to reduce extra cognitive load that doesn't contribute to learning. We will be monitoring and reporting upon the implementation of this project during the semester.

3 Evidence of Teaching Effectiveness

3.1 Awards & Recognitions

I have won two awards for my teaching while a graduate student. Below are their descriptions, with the reward letters in the Appendix.

Edward Peck Curtis Awards for Excellence in Teaching by a Graduate Student

The Edward Peck Curtis Awards for graduate student teaching are given to a small number of full-time graduate students who have a role in undergraduate education. Recipients have assisted in undergraduate instruction, and have had significant face-to-face interaction with undergraduates in the classroom or laboratory.

Winners are selected by the vice provost and University dean of graduate studies based on nominations from individual departments or undergraduate student groups.

Donald M. and Janet Barnard Fellowship

The Donald M. and Janet C. Barnard Fellowship was created for PhD students in one of the University of Rochester's engineering or natural science disciplines. The goal of this fellowship is to recognize outstanding achievement for one of our graduate students in these fields, as evidenced through their coursework and dissertation research work.

The fellowship provides a top-off to the student's existing stipend, as well as a tuition award, and is for a one-year duration.

Each program is allowed to nominate one PhD student in their second or third year of graduate study.

All nominations are evaluated, and the awardees selected by the dean of graduate education. Evaluation is based on documented record of outstanding academic and research achievement as well as evidence of leadership; mentoring and teaching accomplishments; engagement with the field; and any outreach to the community.

3.2 Student Evaluations

I have recieved Teaching Assistant and Instructor Evaluations from students for most courses. The University has removed the previous insentive structure to encourage participation, so response rates are often low.

As an instructor, I've averaged the key results in the following table. The scale is 1 (not at all) to 5 (a lot). The full results are available upon request.

Question	**Average**
What overall rating would you give this instructor?	4.57
Lectures/class discussions were well organized or well managed.	4.54
The instrucor demonstrated sincere respect for students.	4.73

Some selected student comments are

- Donovan is great at explaining difficult concepts in a way that is easy to follow along to and was very good at teaching these topics in calculus.
- He did an amazing job at teaching us, I wish he was teaching me my future math classes
- Donovan did a great job covering the content in this course. He broke things up in a way that was easy to understand and did well at explaining difficult topics while allowing us to practice concepts as well.

A Appendix: Supplementary Materials

A.1 Teaching Award Letters

ARTS, SCIENCES & ENGINEERING

Office of the Dean

Nick Vamivakas
Dean of Graduate Education and Postdoctoral Affairs

UNIVERSITY of ROCHESTER

April 8, 2024

Donovan Snyder
Math

Dear Donovan,

It is a great pleasure to let you know that you have been selected for the Arts, Sciences and Engineering Donald M. and Janet C. Barnard Fellowship. Congratulations! Your strong research record as well as your clear commitment to mentoring, outreach and service to your department and your field are exceptional. I look forward to hearing more about your accomplishments as you continue in your PhD program.

This fellowship provides a stipend of $3,000 for the 2024-2025 academic year. Instructions will be provided to your department administrator to facilitate the stipend payment to begin in September.
Congratulations again!

Sincerely,

Nick Vamivakas
Dean of Graduate Education and
Postdoctoral Affairs Arts, Sciences, &
Engineering

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585.275.4153 · ASEGEPA@rochester.edu · www.rochester.edu/college/gradstudies

Richard Libby, PhD
Interim Vice Provost and
University Dean of Graduate Education

UNIVERSITY of ROCHESTER

February 13, 2025

Dea Donovan,

I am delighted to inform you that you have been selected as one of the winners of the University of Rochester 2025 Edward Peck Curtis Award for Excellence in Teaching by a Graduate Student. This award, established by Edward Peck Curtis in 1984, recognizes graduate students who excel in advancing the teaching mission of the University by providing highly skilled and innovative instruction to our undergraduate students. I was thoroughly convinced by the nomination submitted by the faculty that you are an outstanding educator with a bright future. Congratulations!

The winners will be honored at an exclusive in-person event as part of Graduate Student Appreciation Week. Please join us for the awards reception on April 8, 2025, from 4:30 to 6:00 pm. Kindly RSVP using the provided link, (you will have to be on a campus network or VPN) and feel free to bring a guest along to share in this special occasion. We're excited to extend an invitation to Paul Funkenbusch, the individual who nominated you for this esteemed award, to join in your celebration together.

The award includes an honorarium of $1250. This will be paid by a check from the University Graduate Education office. To receive the check, please complete the attached W9 form and email it to: **UnivGradEducation@UR.Rochester.edu.** If you have already completed a W9 form at UR that includes your mailing address, you may email that copy. We don’t have access to forms submitted to other departments.

Once again, my hearty congratulations on winning this prestigious award.

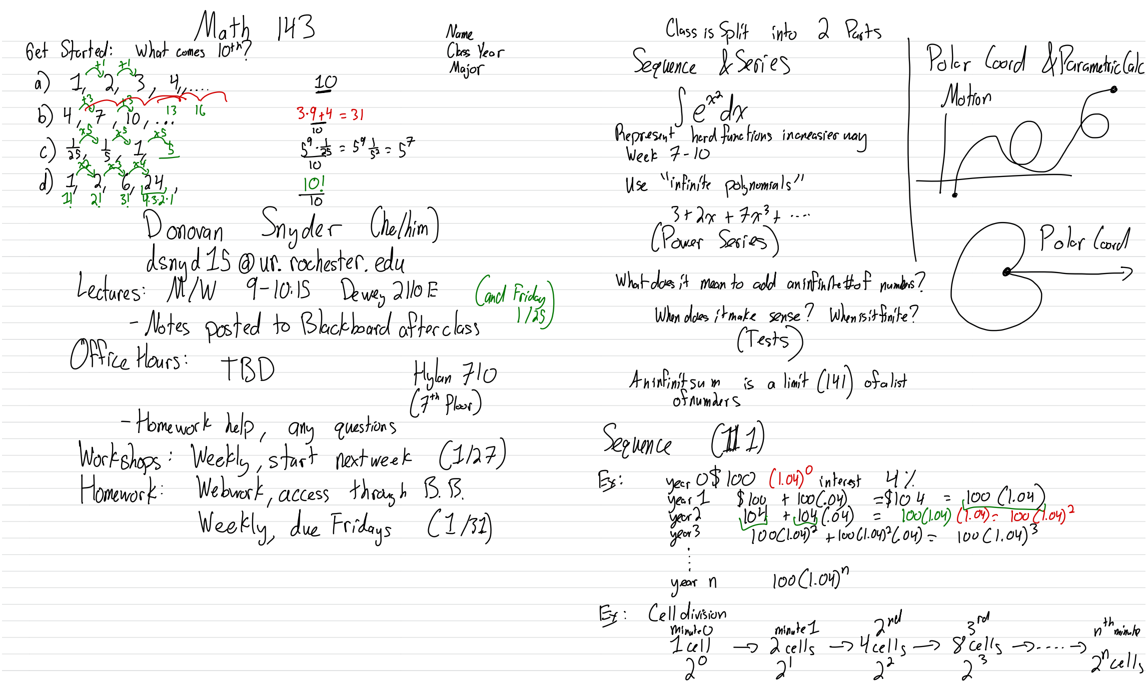
Sincerely,

Richard Libby, PhD
Interim Vice Provost and Dean of University Graduate Education

Box 270015 · Rochester, NY 14627-0015
585.275.3540 · richard_libby@urmc.rochester.edu

A.2 Sample Lecture Notes

Lecture Notes from Day 1 of MATH 143, Calculus III



Math 143

Name
Class Year
Major

Get Started: What comes 10th?

a) 1, 2, 3, 4, ... 10

b) 4, 7, 10, ... 13 16 $3\cdot 9+4=31$

c) $\frac{1}{25}, \frac{1}{5}, 1, 5$ $5^9\cdot\frac{1}{25}=5^9\frac{1}{5^2}=5^7$

d) $\frac{1}{1!}, \frac{2}{2!}, \frac{6}{3!}, \frac{24}{4\cdot3\cdot2\cdot1}$ $\frac{10!}{10}$

Donovan Snyder (he/him)

dsnyd15@ur.rochester.edu

Lectures: M/W 9-10:15 Dewey 2110E (and Friday 1/25)

- Notes posted to Blackboard after class

Office Hours: TBD Hylan 710 (7th Floor)

- Homework help, any questions

Workshops: Weekly, start next week (1/27)

Homework: Webwork, access through B.B.

Weekly, due Fridays (1/31)

Class is Split into 2 Parts

Sequence & Series | Polar Coord & Parametric Calc

$\int e^{x^2}dx$ Motion

Represent hard functions in easier way

Week 7-10

Use "infinite polynomials"

$3+2x+7x^3+\cdots$

(Power Series) Polar Coord

What does it mean to add an infinite # of numbers?

When does it make sense? When is it finite?

(Tests)

An infinite sum is a limit (141) of a list of numbers

Sequence (11.1)

Ex: year 0 \$100 $(1.04)^0$ interest 4%

year 1 \$100 + 100(.04) = \$104 = 100(1.04)

year 2 104 + 104(.04) = 100(1.04)(1.04) = $100(1.04)^2$

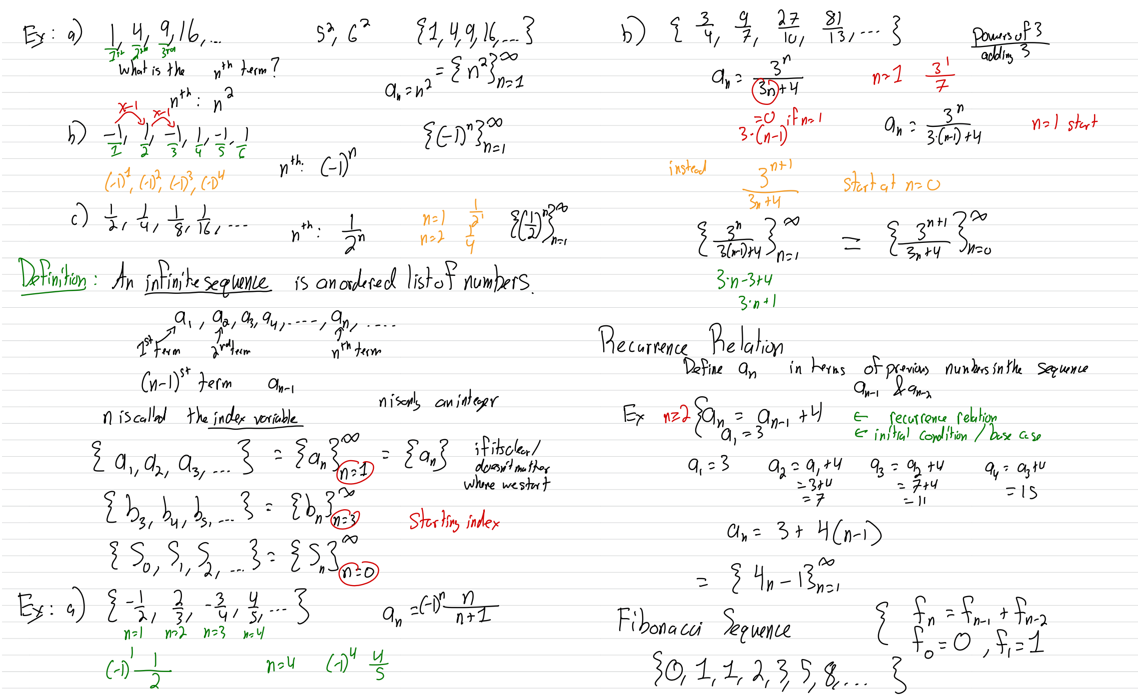
year 3 $100(1.04)^2+100(1.04)^2(.04) = 100(1.04)^3$

⋮

year n $100(1.04)^n$

Ex: Cell division

minute 0: 1 cell 2^0 → minute 1: 2 cells 2^1 → 2nd: 4 cells 2^2 → 3rd: 8 cells 2^3 → ... → nth minute: 2^n cells



Ex: a) 1, 4, 9, 16, ... $5^2, 6^2$ $\{1,4,9,16,\ldots\}$

1st 2nd 3rd What is the nth term?

nth: n^2 $a_n=n^2=\{n^2\}_{n=1}^{\infty}$

b) $-\frac{1}{1}, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}, \frac{1}{6}$ $\{(-1)^n\}_{n=1}^{\infty}$

$(-1)^1, (-1)^2, (-1)^3, (-1)^4$ nth: $(-1)^n$

c) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ nth: $\frac{1}{2^n}$ n=1 $\frac{1}{2}$ n=2 $\frac{1}{4}$ $\{(\frac{1}{2})^n\}_{n=1}^{\infty}$

Definition: An infinite sequence is an ordered list of numbers.

$a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$

1st term, 2nd term, nth term

$(n-1)^{st}$ term a_{n-1}

n is always an integer

n is called the index variable

$\{a_1, a_2, a_3, \ldots\} = \{a_n\}_{n=1}^{\infty} = \{a_n\}$ if it's clear / doesn't matter where we start

$\{b_3, b_4, b_5, \ldots\} = \{b_n\}_{n=3}^{\infty}$ Starting index

$\{S_0, S_1, S_2, \ldots\} = \{S_n\}_{n=0}^{\infty}$

Ex: a) $\{-\frac{1}{2}, \frac{2}{3}, -\frac{3}{4}, \frac{4}{5}, \ldots\}$ $a_n=(-1)^n\frac{n}{n+1}$

n=1 n=2 n=3 n=4

$(-1)^1\frac{1}{2}$ n=4 $(-1)^4\frac{4}{5}$

b) $\{\frac{3}{4}, \frac{9}{7}, \frac{27}{10}, \frac{81}{13}, \ldots\}$ Powers of 3, adding 3

$a_n=\frac{3^n}{3n+4}$ n=1 $\frac{3^1}{7}$

$3\cdot(n-1)$ =0 if n=1 $a_n=\frac{3^n}{3(n-1)+4}$ n=1 start

instead $\frac{3^{n+1}}{3n+4}$ start at n=0

$\left\{\frac{3^n}{3(n-1)+4}\right\}_{n=1}^{\infty} = \left\{\frac{3^{n+1}}{3n+4}\right\}_{n=0}^{\infty}$

$3\cdot n-3+4$

$3\cdot n+1$

Recurrence Relation

Define a_n in terms of previous numbers in the sequence a_{n-1} & a_{n-2}

Ex n≥2 $\begin{cases} a_n = a_{n-1}+4 \\ a_1=3 \end{cases}$ ← recurrence relation ← initial condition / base case

$a_1=3$ $a_2=a_1+4=3+4=7$ $a_3=a_2+4=7+4=11$ $a_4=a_3+4=15$

$a_n=3+4(n-1)$

$=\{4n-1\}_{n=1}^{\infty}$

Fibonacci Sequence $\begin{cases} f_n=f_{n-1}+f_{n-2} \\ f_0=0, f_1=1 \end{cases}$

$\{0, 1, 1, 2, 3, 5, 8, \ldots\}$

Ex: a) $a_n = -3 \cdot a_{n-1}$ $\quad a_1 = 2$ Remove Recurrence

$a_1 = 2 \quad a_2 = -3 \cdot 2 \quad a_3 = (-3)^2 \cdot 2 \quad a_4 = (-3)^3 \cdot 2$

$a_n = 2(-3)^{n-1}$

b) $a_1 = \frac{1}{2} \quad a_n = a_{n-1} + \left(\frac{1}{2}\right)^n$

$a_1 = \frac{1}{2}$

$a_2 = \frac{1}{2} + \left(\frac{1}{2}\right)^2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$a_3 = \frac{3}{4} + \left(\frac{1}{2}\right)^3 = \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$

$a_4 = \frac{7}{8} + \left(\frac{1}{2}\right)^4 = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}$

$a_n = \frac{\text{powers of } 2 - 1}{\text{powers of } 2} \quad a_n = \frac{2^n - 1}{2^n}$

Ex: a) $\left\{\frac{1}{5}, -\frac{1}{7}, \frac{1}{9}, -\frac{1}{11}, \dots\right\}$

$a_n = \frac{(-1)^{n+1}}{5 + 2(n-1)} \quad (-1)^{n+1} = (-1)^{n-1}$

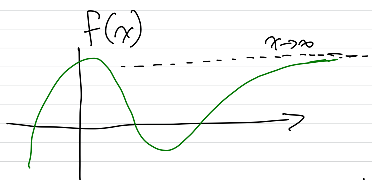
$n = 1 \quad \frac{(-1)^2}{5 + 0} = \frac{1}{5}$

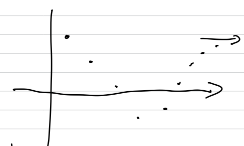
b) $a_1 = -4 \quad a_n = a_{n-1} + 6$

$a_n = -4 + 6(n-1)$

$= 6n - 10$

n=1 start

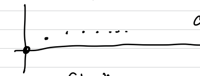




as n gets large, a_n approach some fixed, finit value

Ex: a) $\left\{1 - \left(\frac{1}{2}\right)^n\right\}_{n=0}^{\infty} = \left\{0, \frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \dots\right\}$

$\left\{\frac{2^n - 1}{2^n}\right\}$

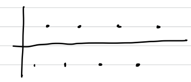


approach 1

$\left(\frac{1}{2}\right)^n \to 0$

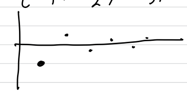
expect $1 - \left(\frac{1}{2}\right)^n \to 1$

b) $\{(-1)^n\} = \{-1, 1, -1, 1, \dots\}$



oscillating
no limit

c) $\left\{\frac{(-1)^n}{n}\right\} = \left\{\frac{-1}{1}, \frac{1}{2}, \frac{-1}{3}, \frac{1}{4}, \dots\right\}$



approaching 0

d) $\{1 + 3n\} = \{4, 7, 10, 13\}$

no limit $\to \infty$

Def: For a sequence $\{a_n\}$, if the terms become arbitrarily close to a finite number L as n becomes large then $\{a_n\}$ is a convergent sequence.

L is the limit of the sequence, $\lim_{n \to \infty} a_n = L$

If $\{a_n\}$ is not convergent, it is a divergent sequence.

limit DNE, $+\infty, -\infty$, oscillating

A.3 Sample Recitation

An Outline for a Recitation for MATH 173, Honors Linear Algebra

1. **10 minutes** (Remember the definition of span from last time)

   (Questions from last time? Homework? Lecture?)

   The **subspace spanned by** a set of vectors $S \subset V$ is the intersection of all subspaces of V containing S.

   (This is always a vector space over the same field as the scalars.)

   And we had a theorem (Theorem 3 in Section 2.2) that this is always the set of linear combinations of the vectors in S: $c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_n\vec{v}_n$

   (S doesn't have to be finite, but any linear combination *must* be)

2. **15 minutes** Example: Describe the vectors spanned by the vectors $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$, and $\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$ over $\mathbb{R}$.

   (Just because we have three vectors doesn't mean we get every vector in $\mathbb{R}^3$.)

   This set of vectors is a vector space, and we don't even need to check the axioms. The same works in reverse:

   Examples: Express as a span of two vectors

   (a) The subset of 2×2 matrices over $\mathbb{R}$

   $$\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} | 2a - c - d = 0 \text{ and } a + 3b = 0 \right\}$$

   (We can find more than two matrices, but try to do it with only two. 2 conditions hopefully means two vectors works.)

   Because it's a linear combination, we know it's a vector space by Theorem 3.

   (b) The subset of polynomials of degree at most 2, $\mathcal{P}_2$, such that $p(2) = 7$.

   (What are the restrictions on the coefficients? Should also be able to get 2 vectors to span)

   (Break)

3. **10 minutes** Remember the definition of linear independence:

   A set of vectors S is **linearly dependent** if there exists distinct vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n \in S$ and scalars $c_1, c_2, \ldots, c_n$, not all zero, such that

   $$c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_n\vec{v}_n = 0$$

   Otherwise, S is **linearly independent**.

   (A particular linear combination (with no non-zero coefficients) can get you zero. There's "overlap")

   (Questions from the in class examples?)

4. **5 minutes** Example that is not $\mathbb{R}^n$: In $\mathcal{P}_2$, is the set $\{1 + x, 1 - x\}$ linearly independent?

   (What is the zero vector in $\mathcal{P}_2$?)

   (What does it mean to show a polynomial is equal to the zero polynomial? Coefficients!)

5. **20 minutes** (Going from a lin indep set to a basis, or a dependent set to a one.)

   Work in $\mathbb{R}^3$ for now. Any set of just one vector is linearly independent, unless it's the zero vector.

$$\left\{\begin{pmatrix}1\\2\\1\end{pmatrix}\right\}$$

   is linearly independent. For our purposes, we can put another vector in the set, in two different ways. (have them choose the vectors)

   Add a vector in the span of the first:

$$\left\{\begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}-3\\-6\\-3\end{pmatrix}\right\}$$

   Add a vector not in the span of the first:

$$\left\{\begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}0\\-1\\0\end{pmatrix}\right\}$$

   (Which set is linearly independent? Why?)

   So we want a linearly independent set, so lets take the right one and repeat the process. (have them choose third vector)

   Add a vector in the span of the first two:

$$\left\{\begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}0\\-1\\0\end{pmatrix}, \begin{pmatrix}2\\2\\2\end{pmatrix}\right\}$$

   Add a vector not in the span of the first:

$$\left\{\begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}0\\-1\\0\end{pmatrix}, \begin{pmatrix}4\\0\\0\end{pmatrix}\right\}$$

   Again, we don't want the first set. The second set is three vectors in $\mathbb{R}^3$. Can we get every vector in $\mathbb{R}^3$ as a linear combination of these three? (Try it. Yes we can.)

   So we've constructed a basis! (This is the proof of Theorem 5 in Section 2.3)

   (The parameterization we were doing before gives us, if we do it right, a linearly independent set!)

6. **10 minutes** Last time we found the set

$$\left\{\begin{pmatrix}x\\y\\z\end{pmatrix} | x+y+z=1\right\}$$

   With operations

$$\begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix} \oplus \begin{pmatrix}x_2\\y_2\\z_2\end{pmatrix} = \begin{pmatrix}x_1+x_2-1\\y_1+y_2\\z_1+z_2\end{pmatrix} \qquad c\odot\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}cx-c+1\\cy\\cz\end{pmatrix}$$

   Was a vector space. (Do you remember the zero vector? the additive inverse?)

   Find a basis by parameterize the set (according to the operations!) and double check it is linearly independent and spans.